A study of the behaviour of a thin sheet of moving liquid

By D. R. BROWN

The British Iron and Steel Research Association, South Wales Laboratories, Swansea

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An investigation has been carried out on the behaviour of a thin sheet of liquid in connexion with the new method of lacquer application known as 'curtain coating'. A method is described for measuring the velocity of the sheet and an equation deduced for calculating this for a liquid of any viscosity. The minimum liquid flow rate required to maintain a stable sheet is discussed, and some effects of the impingement of the sheet on a rapidly moving surface are described.

1. Introduction

When considering the possible methods of applying protective organic coatings at high speed to continuous steel or tinplate strip, the relatively new process of curtain coating appears very attractive. This process as used at present, for example in the furniture trade, consists of pumping the coating material from a supply tank to a precision built head which has a narrow slot along its lower face; the liquid passes through this slot and is formed into a continuous sheet or curtain which falls on to the work pieces as they travel underneath. The coating weight applied varies with the speed of travel of the article through the curtain. For further details of the process see Stein & Podmore (1959) or Bardin (1958).

It was observed that in order to maintain a stable curtain, the rate of flow of liquid through the slot must be above a certain minimum value, hence tending to limit the process to application of thick coatings at reasonable speeds. One objective of this investigation was to determine the physical factors on which the minimum flow rate depends, as well as to study the general behaviour of a liquid in this rather unusual form of a thin moving sheet.

Some interesting work on the disintegration of liquid sheets has recently been carried out by Taylor (1959) and also by Dombrowski & Fraser (1954) and Fraser, Eisenklam & Dombrowski (1957), the latter in connexion with the production of efficient sprays for use in chemical and combustion engineering and agriculture.

2. Experimental technique

The investigation fell naturally into three parts; a study of the behaviour of the liquid in the slot, in the curtain itself, and finally as it impinges on a rapidly moving surface. To study these an adjustable slot of 12 cm length was constructed and a system for pumping liquid through it built up as shown in figure 1. The arrangement was such that the curtain could either fall on to a stationary trough conveniently shaped to allow the liquid to be collected in a flask, or on to the

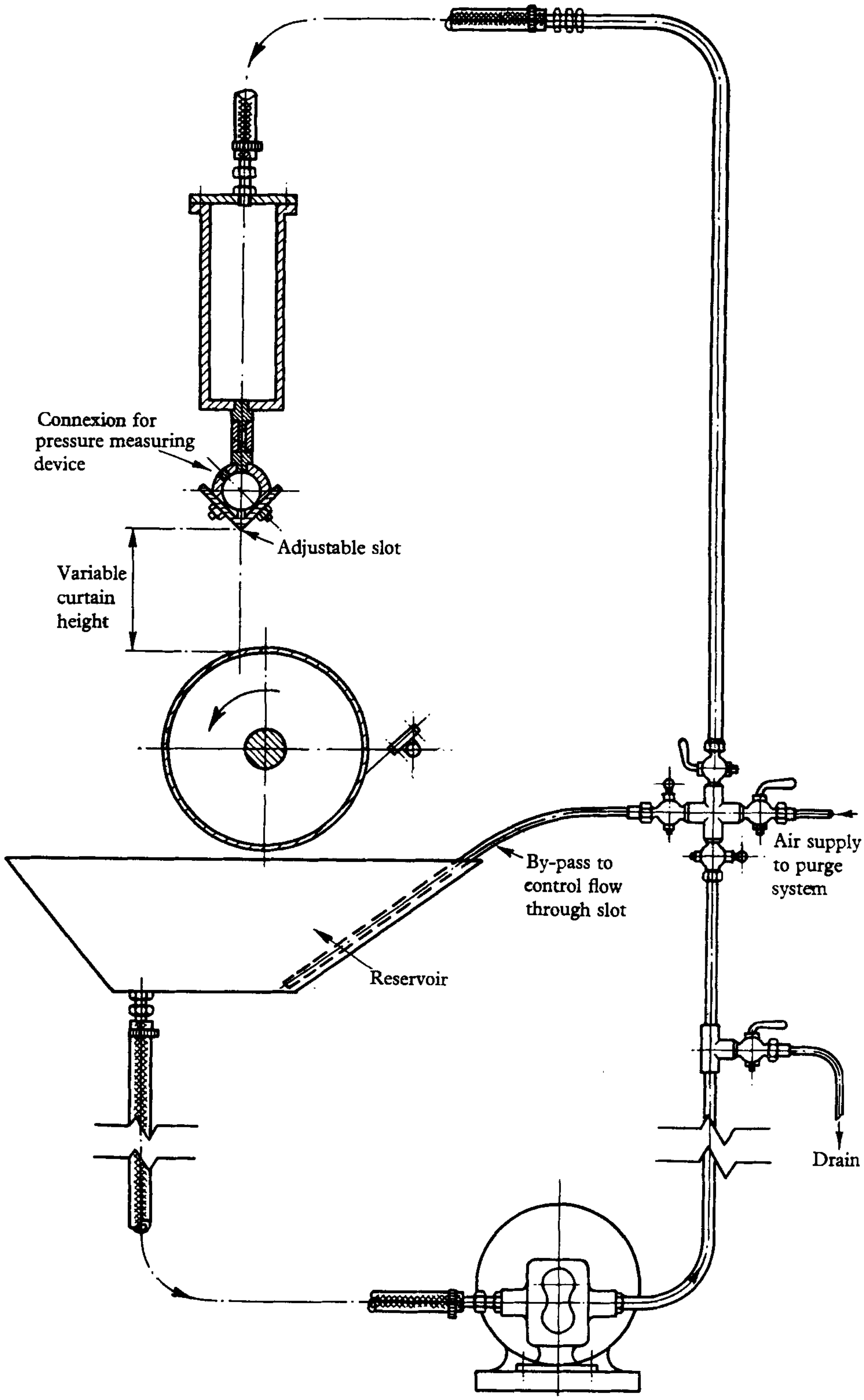


FIGURE 1. Apparatus for producing the liquid curtain.

surface of a roll which could be rotated at peripheral speeds up to 600 cm/sec, and fitted with a scraper blade to remove the liquid.

The liquid used in this work was a lacquer with high solids content, which was thinned to the required viscosity with a high boiling point solvent such as diacetone alcohol. By use of this material a 10:1 range of viscosities was conveniently available. The viscosity was measured by a Ferranti rotation viscometer reading directly in poise and the surface tension by a Cambridge Du Noüy tensiometer calibrated in dyn/cm. As the density of the liquid, determined by a hydrometer, was approximately unity, no more precise measurements were considered necessary and this value was assumed throughout.

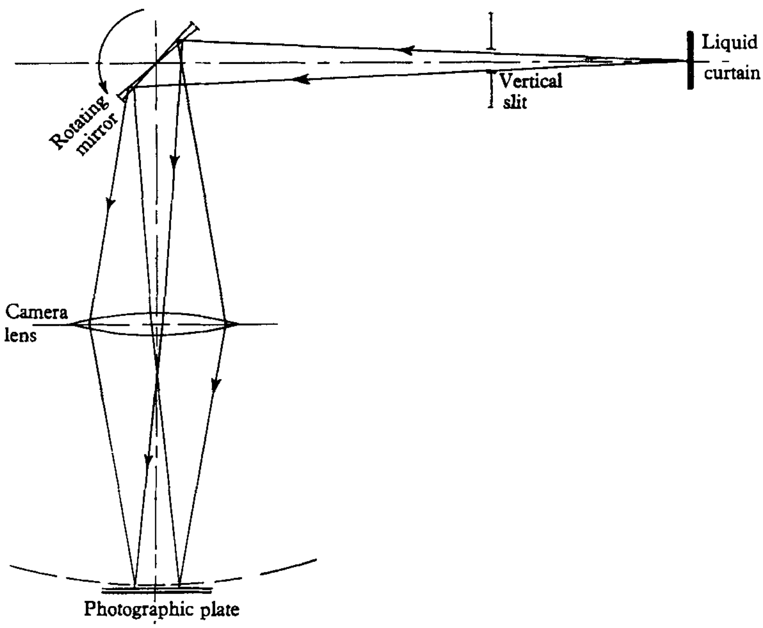


FIGURE 2. Optical system for the velocity measurements.

The flow rate and pressure drop across the slot for various slot widths and liquid viscosities were measured, and also the velocity of the liquid flowing in the curtain under various conditions. Some observations of the behaviour of the liquid as it impinged on the moving roll surface were recorded photographically.

The flow rate was determined by weighing the liquid collected in a weighed flask in a measured time, and the pressure drop by means of a Bourdon gauge calibrated against a mercury manometer, or for the lower pressures by means of a water manometer. The slot width was determined using a feeler gauge. The method used to measure the velocity of the liquid in the curtain was to photograph small air bubbles carried down with the curtain via a rotating plane mirror whose axis of rotation was in a vertical plane parallel with the curtain. In this way curves were obtained on the photographic plate, and from their slope the velocity of the bubble was calculated. This optical system is shown diagrammatically in figure 2 and typical photographic traces, which are effectively graphs of distance against time, are shown in figure 4, plate 1. The mirror was very carefully mounted on a platform running in ball-races and driven at approximately 4 r.p.m. by a geared motor, the essential object being to obtain a smooth rotation at constant speed. Attached to this platform was a cam arranged to trigger a relay which operated the camera shutter at the appropriate instant. The bubbles were illuminated by two photoflood lamps carefully arranged so as to produce traces that were as clear as possible, and in order to reduce the distortion on the plates due to the curved path of the image (as can be seen from figure 2) the object and image distance were made as large as possible compatible with the supplementary camera lenses available. By ensuring that the mirror and camera were correctly set up and aligned, a horizontal reference line was also traced on the plate from a small suitably illuminated object. Measurements were made directly from the plates using a Cambridge Universal Measuring Machine. As a check on the method, some photographs were taken of a $\frac{1}{16}$ in. steel ball-bearing freely falling from rest in air, an example of these being shown in figure 3, plate 1. The velocities at various depths obtained from the slopes of these traces were in close agreement with calculated values.

3. Results and discussion

3.1 *Properties of the liquid*

The density ρ of the liquid was 0·98 g/cm³ and is taken as unity for the purpose of the following calculations. The surface tension T was measured to be 30 dyn/cm. As explained above, the viscosity η was varied over a wide range during the experiments.

3.2 *Pressure and flow-rate measurements*

For the range of viscosities, the pressure drops p required to achieve certain mean velocities u of the liquid through the slot (depth L) for various gap widths b were found to be in good agreement with values calculated from the following equation (see Lamb 1932, p. 582), which applies to the steady flow of liquid between two parallel planes:

$$p = \frac{12\eta L u}{b^2}. \tag{1}$$

In the present case $L = 0\cdot9$ cm. Expressing the velocity in terms of the mass flow rate Q per unit span in g/sec per cm, and writing p' for the pressure measured in cm of mercury, we have

$$p' = 8\cdot1 \times 10^{-4}\frac{\eta Q}{\rho b^3}, \tag{2}$$

with η in poise and b in cm, and with $\rho \doteqdot 1$.

Thus, knowing the dimensions of the slot, assuming it has parallel sides, we can obtain the flow rate directly from the pressure drop across it for a liquid of any viscosity and density. The presence of the curtain does not significantly affect the behaviour of the liquid flowing through such a slot.

3.3 *Velocity measurements*

Some traces obtained by the rotating mirror method are shown in figures 3 and 4, plate 1. Figure 3 shows the trace of a freely falling ball-bearing, and figure 4 typical bubble traces in curtains of low and high viscosity liquid, the slot velocity in these latter cases being similar. Comparison of these photographs shows that with a liquid of low viscosity the velocity at a given depth is approximately that which would be expected from free fall under gravity, but for higher viscosities it is lower.

This difference at higher viscosities is shown more clearly in table 1 where the velocity as measured at a depth of 5 cm below the slot is compared with that calculated assuming free fall with an initial velocity equal to that of the liquid in the slot (the mean slot velocities were calculated from the flow rates and slot widths). To determine more generally the magnitude of this viscosity effect, the velocities u at various distances x below the slot were measured from the

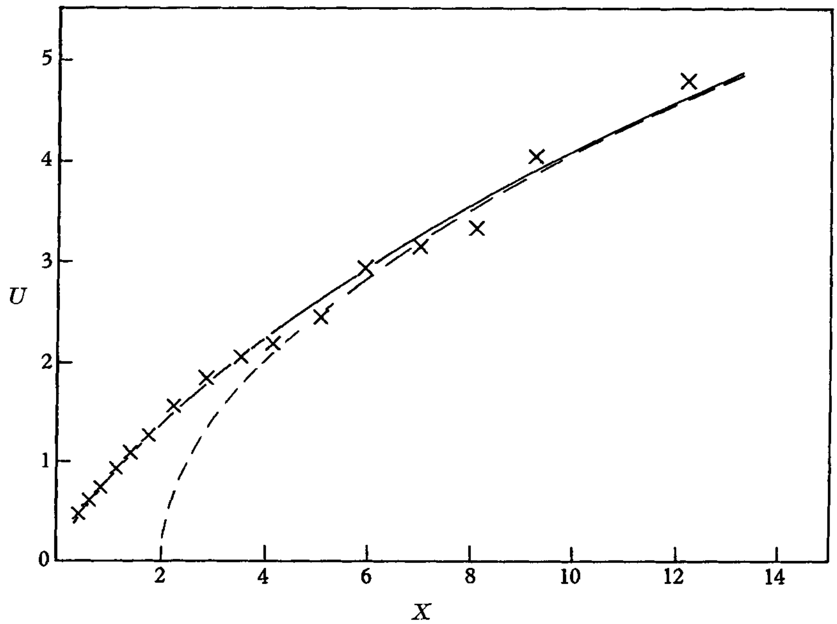


FIGURE 5. Graph of curtain velocity against distance from the slot; ×, experimental results; ----, free-fall parabola.

plates, and the values converted into the dimensionless forms U and X respectively by use of the following expressions (see Appendix):

$$U = \left[\frac{\rho}{4\eta g}\right]^{\frac{1}{3}} u, \tag{3}$$

$$X = \left[\frac{\rho}{4\eta}\right]^{\frac{2}{3}} g^{\frac{1}{3}} x. \tag{4}$$

The curves of U against X obtained from these figures were of the form shown in figure 5, which is for the particular case of a liquid of viscosity 2·6 poise passing through the slot with negligibly small velocity. These curves agree closely with a curve Maruo (1958) obtained by numerical solution of Sir Geoffrey Taylor's equation for a falling sheet of viscous fluid, equation (12) in the Appendix. The curve corresponding to free fall from rest, i.e.

$$U^2 = 2X, \tag{5}$$

was then superimposed on these experimental curves and found to coincide in general with them, for other than small values of X, after displacing its origin to $X = 2$, and making due allowance for the initial slot velocity. This curve is shown as the broken line in figure 5. This means that the velocity attained by the liquid in the curtain after travelling a distance x is that which would have been attained by a body falling freely through a distance $(X-2)$. Converting into cm, we have that this equivalent distance $x_{\text{equiv.}}$ is less than the true distance x travelled by an amount

$$x - x_{\text{equiv.}} = 2(4\eta/\rho)^{\frac{2}{3}} g^{-\frac{1}{3}} \doteqdot 0{\cdot}5\,(\eta/\rho)^{\frac{2}{3}}. \tag{6}$$

Thus the velocity u at a depth x in the curtain for a liquid passing through the slot with a mean velocity u_0 can be expressed by

$$u^2 = u_0^2 + 2g\,[x - 0{\cdot}5\,(\eta/\rho)^{\frac{2}{3}}]. \tag{7}$$

Some values of u calculated from equation (7) for $x = 5$ are given in table 1 and can be seen to agree quite well with the experimental values.

Viscosity (poise)	Slot width (mm)	Mean slot velocity (cm/sec)	Velocity at 5 cm below slot (cm/sec): Experimental	Assuming free fall	Calculated from equation (7)	Qu at base of curtain (17·8 cm long)
1·2	0·6	18	96	101	95	200
1·5	0·6	56	110	114	108	645
1·95	0·2	38	100	106	99	142
2·0	0·2	47	109	110	102	177
2·6	0·6	18	97	101	91	197
2·7	0·3	33	93	104	95	183
2·8	0·3	13	87	100	90	71
3·45	0·6	6	86	99	87	65
3·7	1·5	5	86	99	86	136
5·3	1·5	12	85	100	83	322
9·9	0·6	8	75	99	73	85

TABLE 1

3.4 *Flow rate and stability of curtain in contact with a moving surface*

For a free edge of a thin sheet of moving liquid to exist, equilibrium must be maintained at the edge between the surface tension and the inertia forces of the liquid. Let AB (figure 6) represent such a free edge between a liquid and air. Considering a unit length CD of the edge whose thickness is t, let the velocity of the liquid at the boundary in the direction normal to AB be u. The mass of liquid arriving at CD per second is Q, and its momentum $\rho u^2 t$ (since $Q = \rho u t$). As this momentum is destroyed at the boundary there is a resultant force $\rho u^2 t$, which for equilibrium must be balanced by the surface tension force; i.e.

$$\rho u^2 t = 2T, \tag{8}$$

or

$$Qu = 2T. \tag{9}$$

These equations give the limiting condition for 'stability' at any point in the curtain, in the sense that if a free edge appears by the formation of a hole in the curtain, then if Qu is greater than $2T$ such a hole will not grow but will be carried away with the curtain, whereas if Qu is less than $2T$ it will grow and disrupt the curtain.

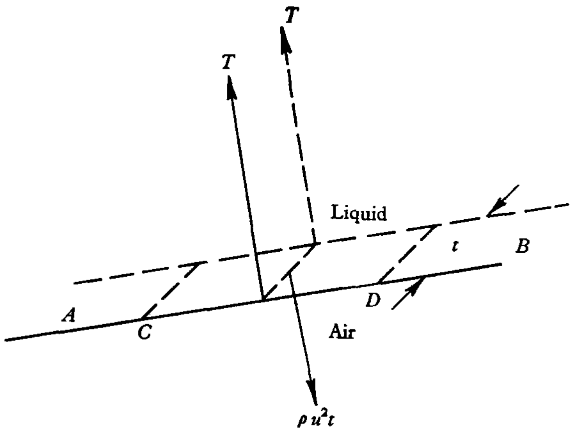


FIGURE 6. Forces at free edge of liquid sheet.

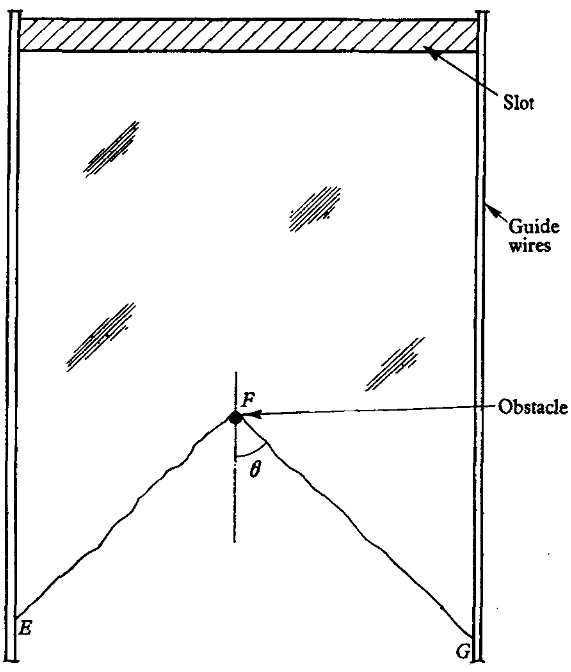


FIGURE 7. Breaking of the curtain round an obstacle.

The theory may be illustrated by introducing an obstacle such as a rod into the curtain and making the liquid flow past it as indicated in figure 7. The component of the velocity, across unit length of the boundary EFG, at any point is $u \sin\theta$ (where u is the velocity in the vertical direction); and hence, from equation (9),

$$\sin\theta = 2T/Qu.$$

In the limit, i.e. when $\theta = 90°$, we have again $Qu = 2T$.

That Qu should be greater than $2T$ near the line of impingement of the curtain with some object is illustrated by the figures in the last column of table 1; the runs in this set with viscosities 2·8, 3·45 and 9·9 poise being with the flow rate near the minimum value ($2T = 60$). For the curtain to be completely stable, that is, free of any tendency for holes to appear spontaneously, Qu must everywhere be greater than $2T$; it was found, however, that a curtain in which Qu was less than $2T$ over a *limited* region near the slot could apparently remain intact, though clearly any hole formed in this metastable region will grow and break the curtain.

By allowing the curtain to fall on to the rotating roll, it was found that under suitable conditions a continuous film could be uniformly laid down on a rapidly moving surface. These conditions are as follows.

(i) For a moderate length curtain such as would be employed industrially, the value of Qu should be greater than $2T$ near the line of impingement of the curtain with the moving surface, since otherwise the disturbance of the flow at the point of impingement will break the curtain.

(ii) The curtain should be protected from air currents carried along with the moving surface. Figure 8 (*a*), plate 2, shows the film being disturbed by such an air current, and figure 8 (*b*) how this can be obviated by the presence of a screen held in contact with the surface immediately behind the curtain. The sharpness of the line of contact of the curtain with the moving surface is remarkable, as can be seen in figure 8 (*b*), and it seemed little affected by the speed of the surface up to 600 cm/sec; at lower surface speeds the air disturbance was less obvious, as might be expected.

(iii) The impingement velocity of the curtain should be above a certain value (of the order of 130 cm/sec) which is apparently independent of the speed of the surface. In figure 8, plate 2, the impingement velocity was 140 cm/sec, but on reducing this to 100 cm/sec, by decreasing the curtain height, it was not possible to obtain a sharp line of contact on the moving surface. This is shown in figure 9, plate 2. Under these conditions the air screen had little effect.

4. Conclusions

(i) A thin curtain of liquid produced by pumping the liquid through a slot suffers from inherent instability in the region near the slot unless the velocity everywhere outside the slot is greater than $2T/Q$. This instability can be avoided by sufficiently increasing the flow rate.

(ii) Small-scale experiments indicate that it is possible to lay down a thin continuous film of liquid on to a rapidly moving surface from a curtain which may be in a metastable condition over a limited region near the slot.

The author is indebted to Sir Geoffrey Taylor for considerable help and encouragement in carrying out this work.

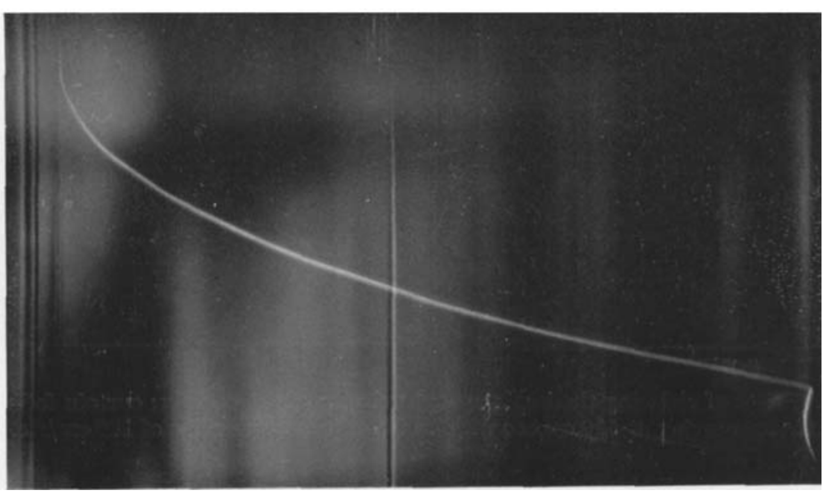
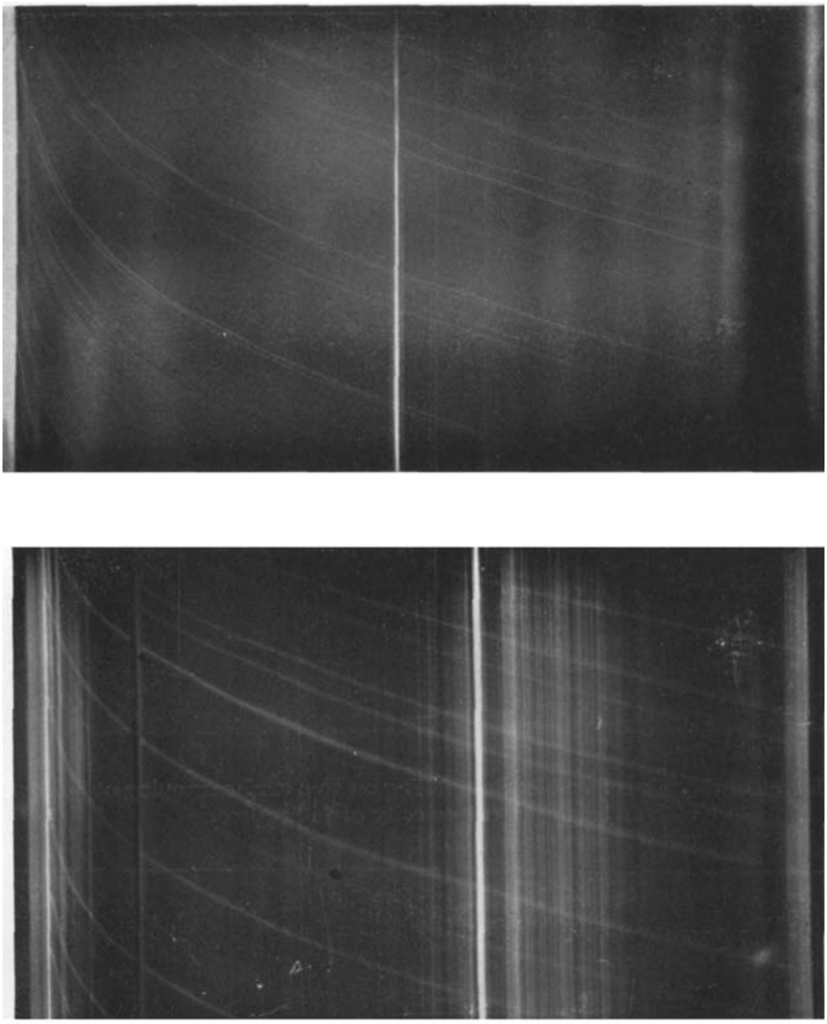
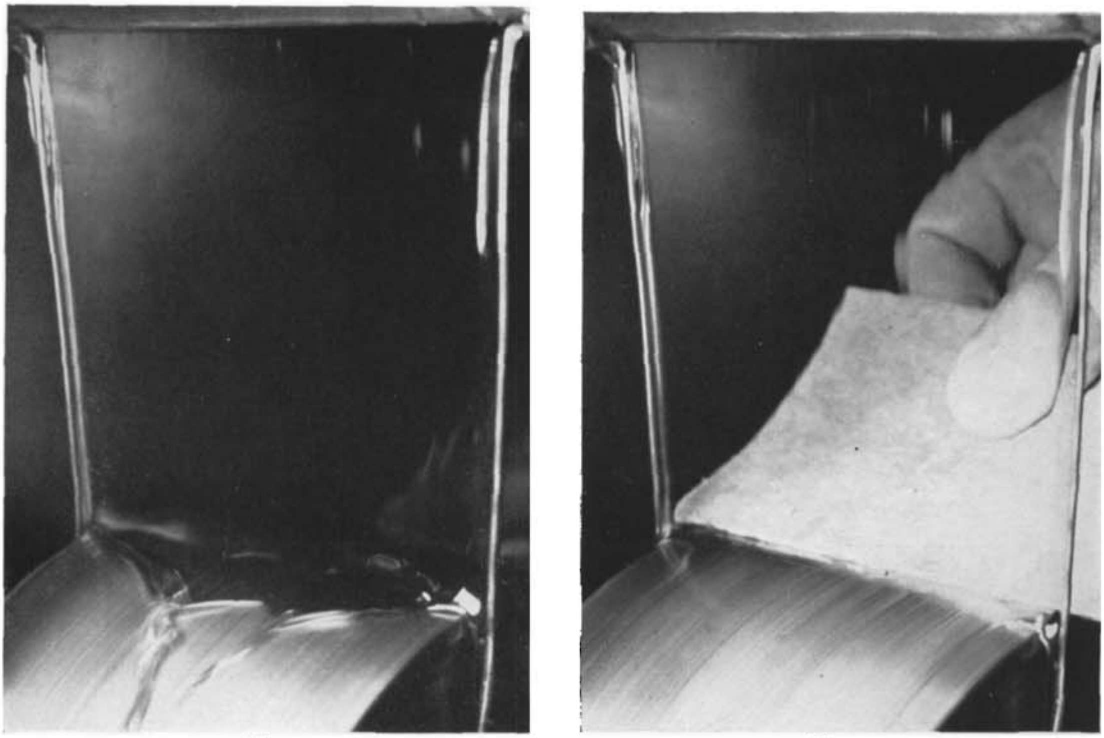


FIGURE 3. Rotating mirror photograph of freely falling ball-bearing.



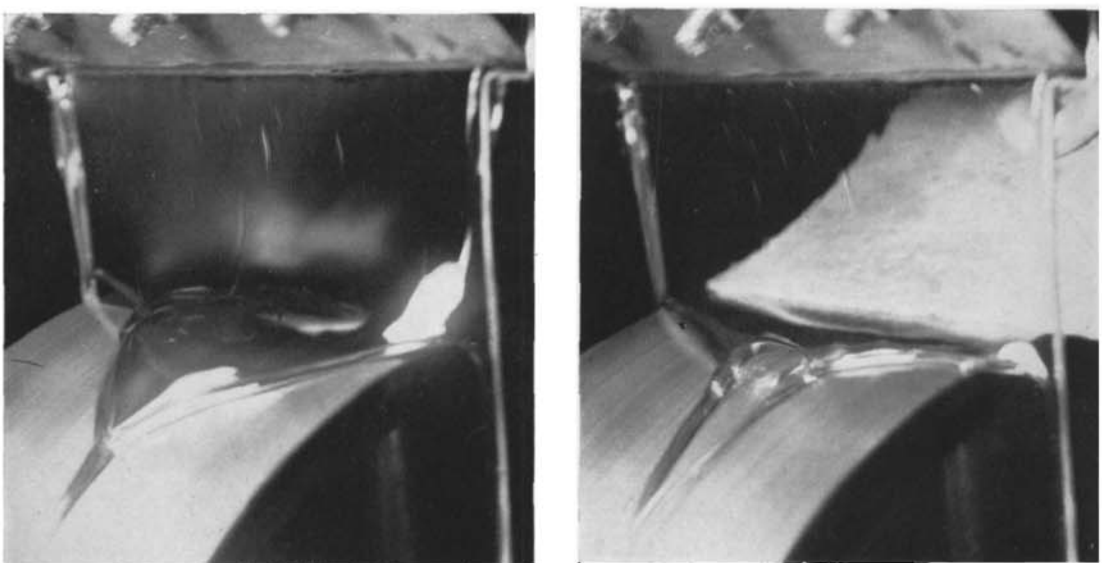
(*a*) (*b*)

FIGURE 4. Rotating mirror photographs of bubbles in a curtain of (*a*) low viscosity liquid, and (*b*) high viscosity liquid.



(*a*) (*b*)

FIGURE 8. Effect of the 'air screen'; curtain falling on to a surface moving at 500 cm/sec with an impingement velocity of 140 cm/sec.



(*a*) (*b*)

FIGURE 9. Effect of reducing the impingement velocity of the curtain; curtain falling on to a surface moving at 500 cm/sec with an impingement velocity of 100 cm/sec.

Appendix

Calculation by Sir Geoffrey Taylor

When the liquid comes down from the slot, the velocity of the flow increases according to the acceleration of gravity. If only the inertia of the liquid is taken into account, the velocity is proportional to the square root of the distance through which the liquid has come down. Since the flux through any section of the jet must be constant, a contraction of the jet is observed which gives a deformation of the liquid resulting in a viscous stress. This stress has an effect upon the velocity of the flow and may be calculated as follows.

Taking the x-axis in the direction of the flow, we have that the stress components are

$$X_x = -p + 2\eta\frac{\partial u}{\partial x}, \quad Y_y = -p + 2\eta\frac{\partial v}{\partial y}, \quad Z_z = -p,$$

where $p = -\frac{1}{3}(X_x + Y_y + Z_z)$ is pressure and it is assumed that $W \equiv 0$ everywhere, because of the guide wires. If the effect of surface tension is neglected and it is assumed that everywhere $u \gg v$, then $Y_y = \text{const.}$, so that, if the atmospheric pressure is p_0, $Y_y = -p_0$. Since

$$\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y},$$

then

$$X_x = -p_0 + 4\eta\frac{\partial u}{\partial x}. \tag{10}$$

Let the thickness of the sheet be t at a depth x from a fixed level. Then

$$\rho u t = Q = \text{const.} \tag{11}$$

The equation of motion becomes

$$Q\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(tX_x) + t\rho g,$$

or, on substitution of (10) and (11) and division by Q,

$$\frac{\partial u}{\partial x} = \frac{4\eta}{\rho}\frac{\partial}{\partial x}\left(\frac{1}{u}\frac{\partial u}{\partial x}\right) + \frac{g}{u}.$$

This equation can be put into non-dimensional form by substituting the following expressions:

$$u = \left(\frac{4\eta g}{\rho}\right)^{\frac{1}{3}} U, \quad x = \left(\frac{4\eta}{\rho}\right)^{\frac{2}{3}} g^{-\frac{1}{3}} X.$$

On rearrangement this gives

$$\frac{d}{dx}\left(\frac{1}{U}\frac{dU}{dX}\right) + \frac{1}{U} - \frac{dU}{dX} = 0. \tag{12}$$

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